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Review of the PhD dissertation
"Embeddings of spaces with multiweighted derivatives and their applications"
by Zhanar Keulimzhayeva

The analysis of function spaces is particularly relevant in several areas of Mathematics such as differential and integral equations. Particularly important are issues such as embedding theorems, direct and inverse trace theorems and the problem of approximation of functions.

The literature in this sense is vast, here we only mention some of the major contributors whose work is related to that of the disseratation under review. Namely, T.I. Amanov, O.V. Besov, V.I. Burenkov, V.P. Ilyin, L.D. Kudryavtsev, P.I. Lizorkin, V.G. Mazya, S.M. Nikolskii, E.D. Nursultanov, M. Otelbaev, H. Tribel, etc.

Among function spaces, the weighted function spaces turn out to be suitable for the analysis of boundary value problems with various types of singularities. The dissertation under review is focused on function spaces with different weighs for different orders of differentiation. In her dissertation, the candidate Zhanar Keulimzhayeva has analyzed the boundary behavior of a function in case the domain is a real semi-axis and the weights can degenerate to 0 at the endpoints 0 and $+\infty$.

The dissertation consists of three chapters.

In the first chapter, following the ideas of L.D. Kudryavtsev, one can interpret the boundary values of a function and of its derivatives at the endpoints 0 and $+\infty$ as follows. One considers a polynomial in a number of linearly independent functions, whose multiweighted derivatives of the n-th order vanish.

If all multiweighted derivatives of the difference of this polynomial and a function from the space tend to zero when the argument tends to zero or infinity, then the function is said to stabilize to the polynomial at zero or at infinity.

Now, if a function from the space of multiweighted derivatives stabilizes to a polynomial at zero or at infinity, then the coefficients of this polynomial are considered to be the "boundary" values of the function at zero or at infinity, respectively.

In this chapter, the candidate proves necessary and sufficient conditions for weight functions in order that each function of the space is stabilized to some unique polynomial, and provides estimates for the rate of stabilization to a polynomial.

Next, the candidate introduces functionals that depend on the boundary values at the singular points and that are equivalent to the norm of the space.

In the second chapter, the candidate proves a weighted estimate for the intermediate multiweighted derivatives and necessary and sufficient conditions for weight functions so that continuous and compact embeddings between spaces of multiweighted derivatives hold.

The candidate also proves conditions on the weight functions in order thaty an inequality of the Nikol'skii-Lizorkin-Kudryatsev type is valid.

Note that the Nikol'skii-Lizorkin inequality on a bounded interval and the Kudryavtsev inequality on an unbounded interval have played an extensive role in the formulation of boundary value problems for degenerate elliptic equations.

Descriptions of the closure of a set of bounded functions in a weighted space of differentiable functions is

an important problem that enables to understand the behavior of a function near the boundary of the domain depending on the singularity type of the weight functions and the results obtained in the dissertation help to formulate boundary value problems for degenerate differential equations.

In the third chapter, the candidate describes the closure of a set of bounded functions in a domain in the space of multiweighted derivatives and proves necessary and sufficient conditions for the weight functions in order that the set of compactly supported functions is dense in the space with multiweighted derivatives.

If the set of bounded functions is not dense in the space, then the whole space is represented as a direct sum of the closure of the set of bounded functions in the domain and of its complement.

The results of this dissertation have resulted into the publication of ten papers: five journal articles (two in a Scopus indexed journal and three in journals that are recommended by the Committee for Control in Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan), and five in proceedings of international scientific conferences.

I believe that the scientific relevance and novelty of the results obtained by the candidate Zhanar Keulimzhayeva on the theme "Embeddings of spaces with multiweighted derivatives and their applications" in her dissertation satisfy all the requirements of PhD dissertations presented to the Doctoral degree and that the author deserves to be awarded the title of Doctor of Philosophy (PhD) in the specialty "6D060100-Mathematics".

Padova, 15 June, 2021

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